

INTRODUCTION TO NUMERICAL NOTATION

INTRODUCTION TO NUMERICAL NOTATIONS

EXPONENTS IN FORMULAS

POWERS OF 10

POSITIVE POWERS OF 10

NEGATIVE POWERS OF 10

CONVERT POWERS OF 10 to PREFIXES

FIGURE 1 (SLIDE EP05AL-S01)

When you measure the length of a board, you may use a standard unit of measurement called the 'foot'. And when you measure the weight of an object, you may use another unit of measurement, such as the 'pound'. In electricity, when you measure voltage, current, and resistance, you use standard units of measurements. The volt, ampere, and ohm are called the basic units of measurements in electrical circuits. There are times, however, when voltage, current, and resistance values are either so large or so small that basic units are too clumsy to be used practically. So, a system of prefixes is used to make the numbers easier to work with.

The purpose of this part of the lesson is to present exponents

Exponents provide a convenient shorthand method for writing or expressing many mathematical operations.

As an example, two cubed may be written to express the number two raised to the third power, thus the number three in the expression would be the exponent. The number three tells what must be done to the number two. The number three, or exponent, says that the number two must be multiplied times itself three times or two times two equals eight. ($2^3 = 2 \times 2 \times 2 = 8$)

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Now it can be seen that the exponent is representative of what is to be done to the base number. The base number, in the above discussion, is the number two. The exponent is the indicator as to what will be done to the base number and the number is three.

Now study the following use of exponents in FIGURE 3.

$$3^2 \text{ (3 squared)} = 3 \times 3 = 9$$

$$5^3 \text{ (5 cubed)} = 5 \times 5 \times 5 = 125$$

$$2^4 \text{ (2 to the fourth)} = 2 \times 2 \times 2 \times 2 = 16$$

$$10^0 \text{ (10 to the zero)} = 1$$

$$10 \text{ (10 to the first power)} = 10$$

$$10^2 \text{ (10 to the second power)} = 10 \times 10 = 100$$

$$10^3 \text{ (10 to the third power)} = 10 \times 10 \times 10 = 1000$$

$$10^6 \text{ 10 to the sixth power} = 10 \times 10 \times 10 \times 10 \times 10 \times 10$$

1,000,000

FIGURE 3 (SLIDE EP05AL-S03)

Exponents may be used in many calculations. The formula in FIGURE 4, show how exponents are used in AC circuits to calculate applied voltage.

$E_A = \sqrt{E_R^2 + E_L^2}$ if $E_R = 10$, and $E_L = 8$, the expression can be written by substituting.

$$E_R^2 = 10^2, \text{ and } E_L^2 = 8^2; E_A = \sqrt{10^2 + 8^2}$$

$$10^2 = 10 \times 10 = 100, 8^2 = 8 \times 8 = 64; E_A = \sqrt{100 + 64}$$

$$100 + 64 = 164; E_A = \sqrt{164} = 12.8V$$

FIGURE 4 (SLIDE EP05AL-S04)

Without the aid of a calculator, some numbers, having larger exponents, can become complicated. Ask the instructor to demonstrate the proper method of solving the problems using the calculator.

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By the use of powers of ten, it will be shown that exponents can be used conveniently to express very large or small numbers. It was discovered that the base number could be any number. In some examples the number ten was used as base number.

If the number ten to the zero power is examined, it may be shown that one time ten to the zero power equals one times one equals one. The number one is expressed as having the decimal point to the right of the one.

One times ten to the first power equals one times ten, which equals ten. The base number, then, is ten. Likewise one times ten squared equals one hundred.

Reversing the process; if one hundred has the decimal to the right of the last zero, and the decimal is moved two places to the left, one hundred becomes one times ten squared.

Study the problems in FIGURE 5.

$$1 \times 10^0 = 1 \times 1 = 1, \text{ or the answer is } 1.0$$

$$1 \times 10^1 = 1 \times 10 = 10, \text{ or the answer is } 10.0$$

$$1 \times 10^2 = 1 \times 10 \times 10 = 100, \text{ or the answer is } 100.0$$

If the number 100.0 was returned to $1 \times 10^2 = 100.0$; the process would have to be done by:

100.0 moving the decimal to the left two places

$$\text{thus } 1 \times 10^2 = 100.0$$

Extremely large numbers can be handled as easily.

6,250,000,000,000,000.0

Move the decimal 18 places to the left and it results in

$$6.25 \times 10^{18}$$

FIGURE 5 (SLIDE EP05AL-S05)

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When using powers of ten as shown in FIGURE 6, numerical values from one to one million can be expressed in positive powers of ten.

Numerical Values	Powers of 10	Expressed Powers of 10
1	10^0	= 10 to the zero power
10	10^1	= 10 to the first power
100	10^2	= 10 to the second power
1000	10^3	= 10 to the third power
10000	10^4	= 10 to the fourth power
100000	10^5	= 10 to the fifth power
1000000	10^6	= 10 to the sixth power

FIGURE 6 (SLIDE EP05AL -S06)

When using the base unit one and multiplying one by ten to the sixth power (1×10^6), it becomes one million. (The decimal point in the number one point zero (1.0), must be moved to the right six places or six zeros must be added to the right).

The simple procedure for converting a whole number, expressed as a power of ten, to its base number, is to write down the number and move the decimal point to the right as many places as indicated by the exponent.

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Study the problem in FIGURE 7, which will demonstrate changing from powers of ten to base numbers.

$$\begin{aligned} 0.345 \times 10^2 &= 0.345 \times 10 \times 10 \\ &= 0.345 \times 100 \\ &= 34.5 \end{aligned}$$

$$0.345 \times 10^2 = \underbrace{0.345}_{\text{move decimal two places to the right}}$$

$$\begin{aligned} 0.345 \times 10^3 &= 0.345 \times 10 \times 10 \times 10 \\ &= 0.345 \times 1000 \\ &= 345. \end{aligned}$$

$$0.345 \times 10^3 = \underbrace{0.345}_{\text{move decimal point 3 places to the right}}$$

FIGURE 7 (SLIDE EPO5AL-S07)

Converting positive numbers to power of ten. To convert a whole number to a power of ten number; move the decimal point to the left from the base to a point indicated by the exponent. For each move of the decimal, one must be added to the exponent.

Study the examples in FIGURE 8. If you do not understand this method, ask your instructor for help.

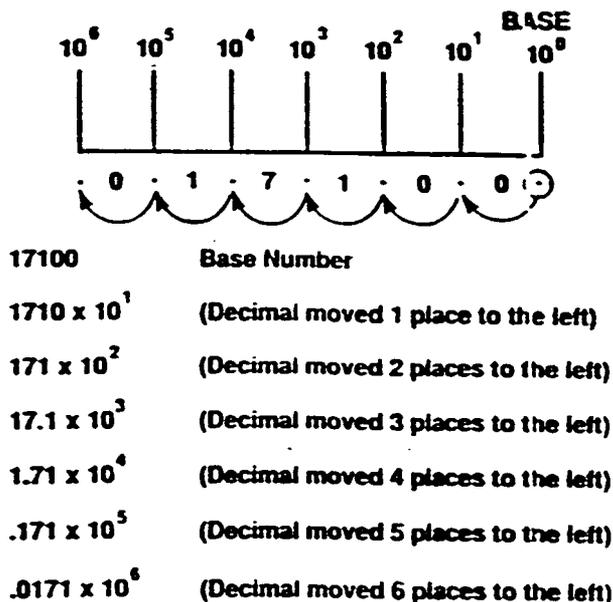


FIGURE 8 (SLIDE EP05AL-S08)

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Converting fractions to negative power of ten. In FIGURE 9; it may be shown that negative powers of ten from the base (10^0) to the right, as the negative exponent becomes larger, that is, becomes more negative.

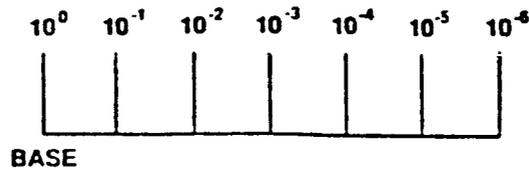
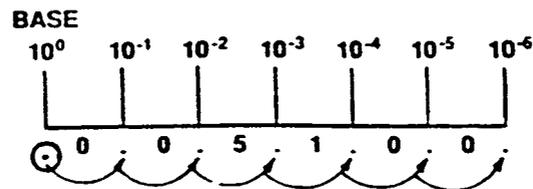


FIGURE 9 (SLIDE EP05AL-S09)

As shown in FIGURE 10, to convert a fraction of a whole number base number, to a negative power of ten, you must move the decimal point right from base, to a point indicates by the exponent. For each move to the decimal point, add one to the exponent.



	Base Number
.0051	
.051 x 10^{-1}	(Decimal moved one place to the right)
.51 x 10^{-2}	(Decimal moved 2 places to the right)
5.1 x 10^{-3}	(Decimal moved 3 places to the right)
51. x 10^{-4}	(Decimal moved 4 places to the right)
510. x 10^{-5}	(Decimal moved 5 places to the right)
5100. x 10^{-6}	(Decimal moved 6 places to the right)

FIGURE 10 (SLIDE EP05AL-S10)

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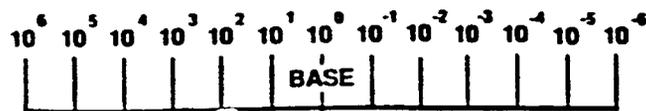
To better understand the procedures in FIGURE 9, use any fractional or decimal number. The fraction must be reduced to the decimal equivalent. The fraction, one hundred ninety-sixth ($1/196$) is equal to point zero five one (0.0051). This number is not only the decimal equivalent but is the base number, and would be located at base or ten to zero power (10^0). Thus the $.0051 \times 10^0 = .0051$. As the decimal moves to the right the exponent increases negatively.

To combine both positive and negative powers of ten on one scale see FIGURE 11.

Using this scale you will notice the base number is still at ten to the zero power (10^0) or 1. Any number may be chosen as base number. The decimal will be the starting point. If the number is less than one, move the decimal to the right, to the first digit greater than zero. The number of digits moved become the negative number for the exponent.

If the number is greater than one, move the decimal to the left to the first digit greater than zero. The number of digits moved to the left to the first digit greater than zero. The number of digits moved will become the positive number for the exponent.

Study FIGURE 11, carefully. This process must be thoroughly understood. If help is needed, now is the time to get the instructor to clear up misunderstandings.



When converting a whole base number to a power of 10, move the decimal point to the left.

Example: $15.100 = 15.1 \times 10^3$

When converting a positive power of 10 to a whole number, move the decimal point to the right.

Example: $1 \times 10^2 = 1.00$

When converting a fraction of a number to a negative power of 10, move the decimal point to the right.

Example: $.0012 = .0012 \times 10^{-3}$

When converting a negative power of 10 to a fraction of a number, move the decimal point to the left.

Example: $73 \times 10^{-4} = .0073$

FIGURE 11 (SLIDE EP05AL-S11)

NOTE: All decimal points will appear at the base if powers of 10 are not being used.

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Addition of numbers may be accomplished when the exponents are the same. If the exponents are not the same, one must be changed to be equal to the other by moving the decimal as previously discussed.

EXAMPLES:

1. $10 \times 10^4 + 5 \times 10^3 + 5 \times 10^3$

$$100 \times 10^3 + 5 \times 10^3 + 5 \times 10^3$$

2. $10 \times 10^{-4} + 5 \times 10^{-3} = 1.00 \times 10^{-3} + 5 \times 10^{-3}$

$$1.0 \times 10^{-3} + 5 \times 10^{-3} = 6 \times 10^{-3}$$

When adding powers of ten, the exponent will remain the same. In the above examples the exponent in the answer as the two numbers to be added.

Subtraction of numbers are treated like addition, the exponents of both numbers must be the same.

EXAMPLES:

When exponents are the same, subtract the two numbers

1. $50 \times 10^3 - 2 \times 10^3$

$$50 - 2 = 48 \times 10^3$$

2. $10 \times 10^{-3} - 2 \times 10^{-2}$

$$1.0 \times 10^{-2} - 2 \times 10^{-2} = 1 \times 10^{-2}$$

Multiplication of numbers using powers of ten, require that the exponents be added taking the sign of the larger exponent.

EXAMPLE:

$$3 \times 10^4 \times 4 \times 10^3 =$$

$$12 \times 10^7$$

If one exponent in the problem is positive and the other exponent is negative, the exponent must be subtracted and the sign of the larger will become the sign of the exponent in the results.

EXAMPLE:

$$3 \times 10^{-4} \times 4 \times 10^3$$

$$12 \times 10^{-1}$$

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Division of numbers using powers of ten, require the sign of the exponent in the denominator be changed. The exponents are added taking the sign of the larger.

EXAMPLE:

$$\frac{4 \times 10^3}{2 \times 10^{-2}} = 2 \times 10^{3+2} = 2 \times 10^5$$

If the sign of the exponents are the same, divide the number and subtract the exponents taking the sign of the larger.

EXAMPLE:

$$\frac{4 \times 10^3}{2 \times 10^2} = 2 \times 10^{3-2} = 2 \times 10^1 = 20$$

Converting powers of ten to prefix values. In electronic measurements some values are either very large or very small. To simplify these values, a letter is used instead of a power of ten. These letters are called prefixes and have a direct relationship to a specific power of ten.

EXAMPLE:

1000 can be converted to 1×10^3 or 1K. K being representative of KILO.

There are numerous prefixes used in electronics, but only six of these are used commonly. The six will be discussed as shown in FIGURE 12.



K = Kilo = $10^3 = 1000$
M = mega = $10^6 = 1000000$
m = milli = $10^{-3} = .001$
 μ = micro = $10^{-6} = .000001$
n = nano = $10^{-9} = .000000001$
p = pico = $10^{-12} = .000000000001$

FIGURE 12 (SLIDE EP05AL-S12)

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Any power of ten can be converted to a prefix value simply by moving the decimal point to the nearest common prefix.

EXAMPLE:

1. 1×10^4 The nearest prefix is K. Move the decimal one place to the right. $10 \times 10^3 = 10\text{K}$
2. 5×10^{-5} The nearest common prefix is MICRO (μ). Move the decimal one place to the right. $50 \times 10^{-6} = 50\mu$.

All rules pertaining to adding, subtracting, multiplying or dividing prefixes will be the same as powers of ten.

EXAMPLES:

$$10\text{K} + 10\text{K} = 20\text{K}$$

$$5\text{m} \times 5\text{K} = 25\Omega$$

$$5\text{M} - 5\text{K} = 4.99\text{M}$$

SUMMARY:

Sometimes basic units are either too large or small to use practically when writing or making computations. When changing from prefix values to basic values, the conversion can be accomplished by multiplying by the prefix value, or shifting the decimal place. When changing from basic values to prefix value, the conversion can be accomplished by dividing the basic value by the prefix value, or by shifting the decimal place. Prefix values can also be converted to powers of ten, which is called scientific notation. When the powers of ten are being used in calculations, certain rules must be followed, according to the operation. When adding powers of ten, the exponents of the numbers to be added must be the same. This same rule applies for subtraction. When multiplying numbers using the powers of ten, add the exponents. In division change the sign of the exponent in the denominator, and proceed as in addition.