# PARALLEL CIRCUITS

So far, you've studied the operating characteristics of one of the basic types of circuits, the series circuit. During this lesson you will learn the characteristics of a second type of circuit known as the parallel circuit. Parallel circuits are used in your home wiring system so that you can turn lights on and off, or plug appliances in and out without interrupting current throughout the house. In other words, each outlet in a house wiring system is in parallel with the other outlets. By the use of Ohm's Law and other simple rules that you will learn in this conference, you will be able to solve parallel circuits problems, just as you did in series circuits.

A parallel circuit may be defined as an electrical circuit having more than one path for current flow.

Resistance in a parallel circuit:

When all the resistors in a parallel circuit are the same value, the equivalent resistance of the circuit is calculated by dividing the number of parallel branches into the value of the one resistor.

As shown in **FIGURE 3**, all three resistors are the same value.

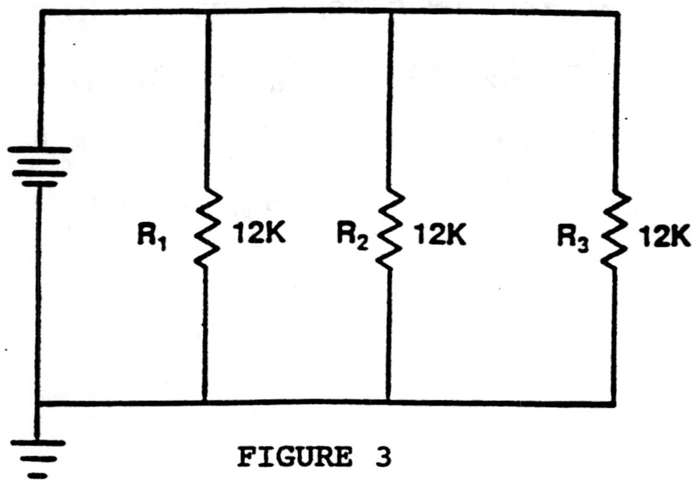


**FIGURE 3**

To find total resistance or equivalent resistance for parallel resistors that are equal, divide one resistor by the number of resistors.

Example:

$$R_T = \frac{R_1}{3} = \frac{12k}{3} = 4k\Omega$$

Thus $R_T$ = Req = 4KΩ
The equivalent resistance of the circuits is 4KΩ.

When a parallel circuit contains two resistors of unequal value the product over the sum method may be used.

It has been experimentally demonstrated that the equivalent resistance of two unequal parallel resistors could be calculated by use of the following formula:

$$Req = \frac{R_1 \times R_2}{R_1 + R_2}$$

In FIGURE 4, we have unequal resistors in parallel.

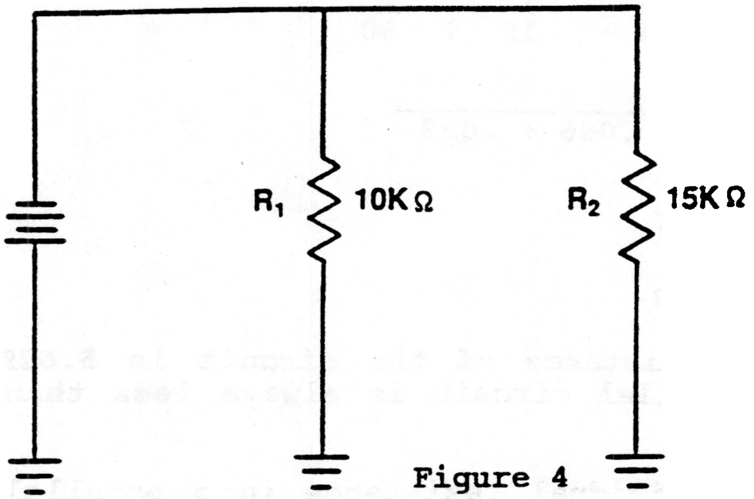


Figure 4

EXAMPLE:

$$R_{eq} = \frac{R1 \times R2}{R1 + R2}$$

$$R_{eq} = \frac{10 \times 15}{10 + 15}$$

$$R_{eq} = \frac{150}{25}$$

$$R_{eq} = 6\Omega$$

Thus, the equivalent resistance is 6 ohms.

When a parallel circuit contains more than two resistors of unequal value, the reciprocal method may be used. As shown in FIGURE 5, three resistors of unequal value can be calculated as follows:

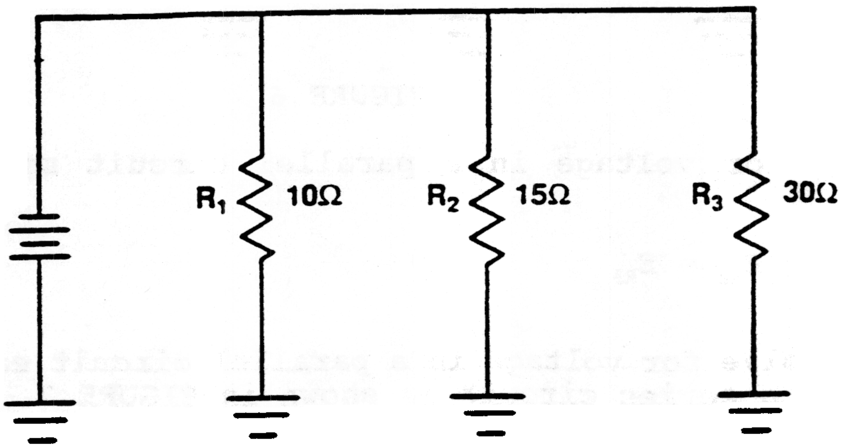


FIGURE 5

EXAMPLE: $R_{ea} = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$

EXAMPLE: $R_{eq} = \dfrac{1}{\dfrac{1}{10} + \dfrac{1}{15} + \dfrac{1}{30}}$

$$\frac{1}{.1 + .066 + .033}$$

$$\frac{1}{.199}$$

$$= 5.02\Omega$$

The equivalent resistance of the circuit is 5.02Ω. The total resistance of a parallel circuit is always less than the smallest branch of resistance.

When measuring individual resistance in a parallel circuit, the resistors must be isolated.

To calculated voltage in a parallel circuit it must be understood that the voltage drops are the same as the voltage applied. This may be seen in FIGURE 6.

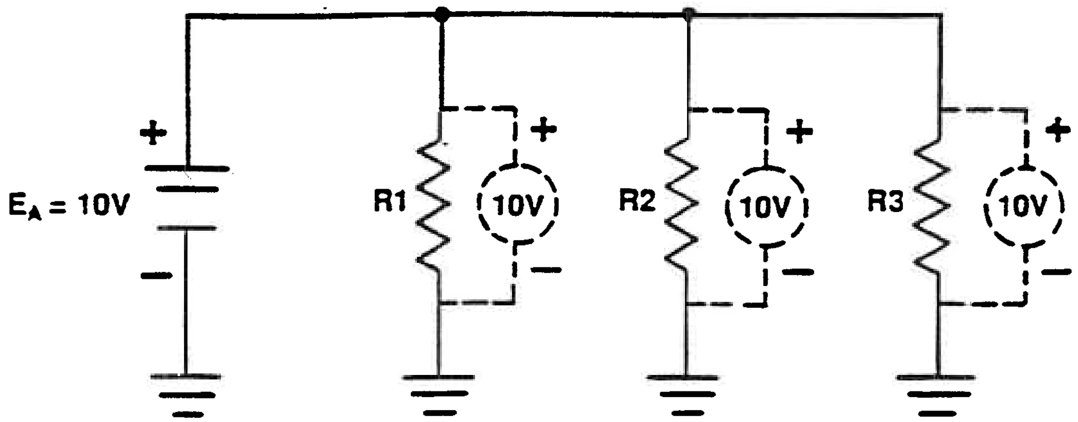


FIGURE 6

The law for voltage in a parallel circuit may be expressed as follows:

$E_A = E_{R1} = E_{R2} = E_{R3}$

To solve for voltage in a parallel circuit each branch must be treated like a series circuit as shown in FIGURE 7.

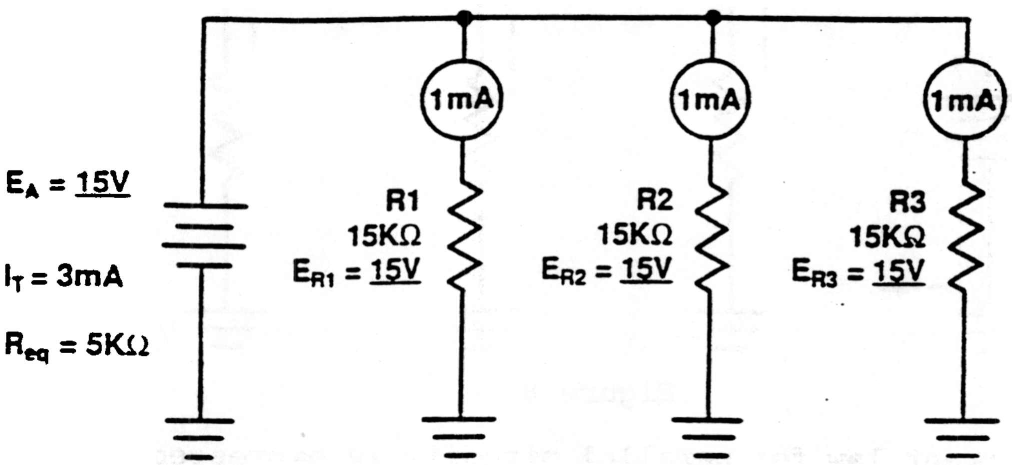


FIGURE 7

EXAMPLE: $E_A = I_T \times R_{eq}$

$= 3ma \times 5K$

$= (3 \times 10^{-3})\ (5 \times 10^{3})$

$10^{0}$

$= 15V$

Applied voltage is 15V.

$E_A = E_{R1} = E_{R2} = E_{R3}$

$E_{R1} = 15V,\ E_{R2} = 15V,\ E_{R3} = 15V$

To calculate current flow in a parallel circuit, Kirchhoff's current law along with Ohms's law must be understood. Kirchhoff's current law says that the total current ($I_T$) of a parallel circuit is equal to the sum of the branch currents. Kirchhoff's Law also states that the current entering a point is equal to the current leaving. Refer to figure 8.

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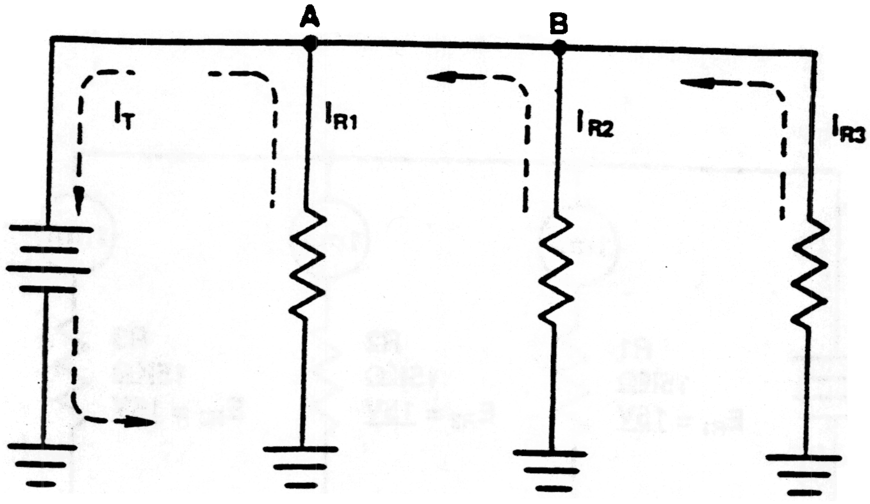


Figure 8

Kirchhoff's current law for parallel circuits is expressed as follows;

$I_T = I_{R1} + {}_{R2} + I_{R3}$

Thus the current entering the ground side of each resistor is equal the current leaving the top side of each resistor.

Current paths in a parallel circuits are called branches.

In a parallel circuit, the total current divides among the individual branches in relation to branch resistance.

In figure 9 it is shown that the sum of the branch currents equals the total current entering and leaving the circuit.

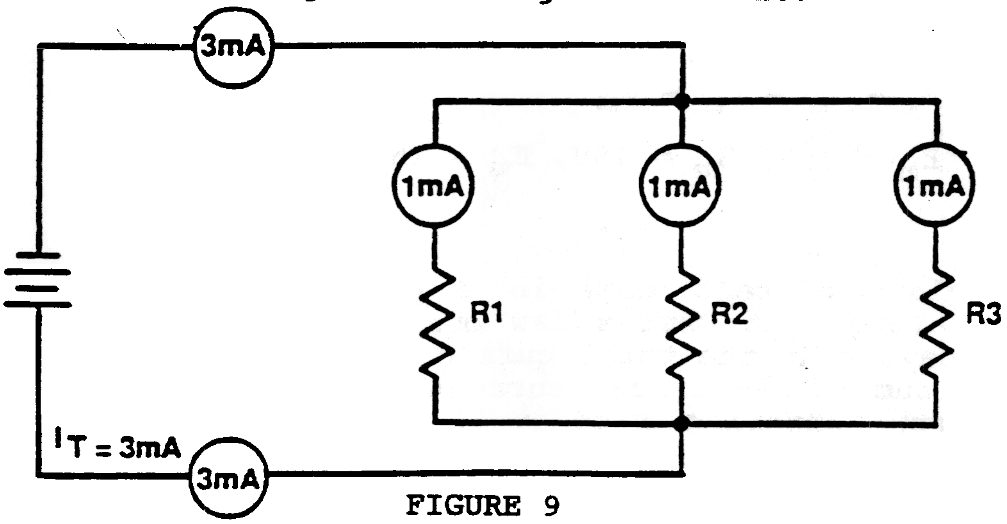


FIGURE 9

EXAMPLE: $I_T = I_{R1} + I_{R2} + I_{R3}$

$I_T = 1\ ma + 1\ ma + 1\ ma$

$I_T = 3\ ma$

## PARALLEL CIRCUITS

Kirchhoff's laws for parallel circuits:

Kirchhoff's Current law: The current entering a junction must equal the current leaving that junction.

Kirchhoff's Voltage Law: In any complete electrical circuit, the sum of voltage drops must be equal the applied voltage.

Solve for unknown currents using the laws for parallel circuits and Ohm's Law, as shown in FIGURE 10.

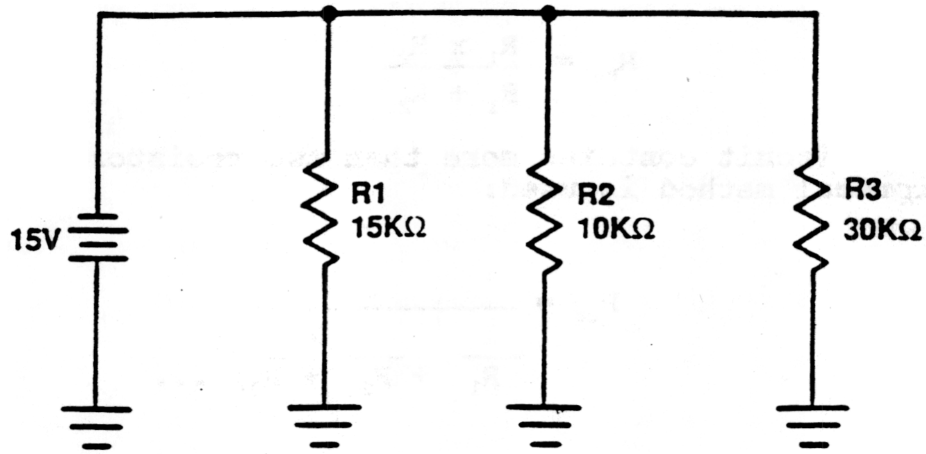


FIGURE 10

EXAMPLE:

$I_{R1} = \frac{E_A}{R_1}$  $I_{R2} = \frac{E_A}{R_2}$  $I_{R1} = \frac{E_A}{R_3}$

$= \frac{15V}{15K}$  $= \frac{15V}{10K}$  $= \frac{15V}{30K}$

$\frac{15}{15 \times 10^3}$  $= \frac{15}{10 \times 10^3}$  $= \frac{15}{30 \times 10^3}$

$= 1 \times 10^{-3}$  $= 1.5 \times 10^{-3}$  $= .5 \times 10^{-3}$

1 ma  = 1.5 ma  = .5 ma

$I_{R1}$ is 1 ma  $I_{R2}$ is 1.5 ma  $I_{R3}$ is .5 ma

Since $IT = I_{R1} + I_{R2} + I_{R3}$

$I_T = 1 \text{ ma} + 1.5 \text{ ma} + .5 \text{ ma}$

$I_T = 3 \text{ ma}$

PARALLEL CIRCUITS

SUMMARY:

During this lesson you have been taught how to calculate resistances, voltages, and currents in parallel circuits. When calculating the equivalent resistance of a parallel circuit, one of the following formulas will be used:

If the parallel circuit contains all resistors of equal value:

$$\frac{R_1}{N \text{ (\# of resistors)}}$$

If the circuit contains two resistors of unequal value, the product over the sum method is used:

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$$

If the circuit contains more than two resistors of unequal value, the reciprocal method is used:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}, \ldots}$$

No calculations are required when voltages in a parallel circuit are needed, because all branch voltages are equal to the applied voltage. Kirchhoff's voltage law for parallel circuits says the voltage across each branch of a parallel circuit is equal to the voltage across every other branch and to the applied voltage. Stated mathematically:

$$E_A = E_{R1} = E_{R2} = E_{R3}, \ldots$$

When calculating the branch currents in a parallel circuit, Ohm's Law can be used since the branch voltage is equal to the applied voltage. Kirchhoff's current law for parallel circuits states total current equals the sum of the currents of the parallel branches. Expressing it mathematically,

$$I_T = I_{R_1} + I_{R_2} + I_{R_3}$$

## PARALLEL CIRCUITS

The final conclusion to the parallel circuit is that it must be reduced to a one-resistor one power supply series circuit, as shown in FIGURE 11.

Circuit "A" Circuit "B" Circuit "C"

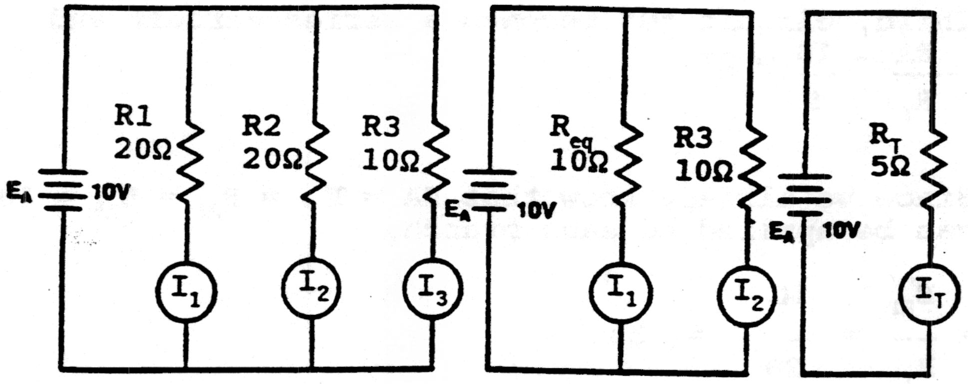


FIGURE 11

This type of problem can be worked 2 ways:

First:

$$R_1 = R_2 = 20$$

$$R_{eq1,2} = \frac{R1}{2} = \frac{20}{2} = 10, \text{ since } R_{eq1,2} = R_3 = 10$$

$$= \frac{R_3}{2} = \frac{10}{2} = 5 \qquad R_{eq} = 5 \text{ ohms}$$

Second:

$$\frac{1}{\frac{1}{\ } + \frac{1}{\ } + \frac{1}{R_3}}$$

$$\frac{1}{\frac{1}{\ } + \frac{1}{20} + \frac{1}{10}}$$

$$\frac{1}{.05 + .05 + .1}$$

$$R_{eq} = \frac{1}{.2} \qquad R_{eq} = 5 \text{ ohms}$$

PARALLEL CIRCUITS

Referring to FIGURE 11, it is readily shown that

$I_T = I_1 + I_2 + I_3$, in circuit "A".

Second, it may also be shown that $I_T = I_1 + I_2$ in circuit "B"

Third, circuit "C" becomes a series circuit and

$$I_T = \frac{EA}{R_T} = \frac{10}{5} = 2A.$$

Since we already know that $EA = E_{R1} = E_{R2} = E_{R3} = 10V$, Ohm's law can be applied to each branch;

$$I_1 = \frac{E_{R1}}{R_1} = \frac{10}{20}$$

$$I_2 = \frac{E_{R2}}{R_2} = \frac{10}{20} =$$

$$I_3 = \frac{E_{R3}}{R_3} = \frac{10}{10} = 1.0A$$

$$I_T = I_1 + I_2 + I_3 = 2A$$

On the basis of the above calculations we can predict what will happen when shorts and opens are introduced to the parallel circuit.

In FIGURE 11, there are a limited set of shorts and opens that can be applied to the circuit.

The total current flow is determined by the total resistance and the applied voltage in the circuit;

EXAMPLE:

$$I_T = \frac{E_A}{R_T}$$

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{\phantom{R_2}} + \frac{1}{R_3}}$$

The applied voltage does not normally change and therefore is not considered a malfunction for these problems.

## PARALLEL CIRCUITS

If $R_T$ increases, $I_T$ will decrease. $R_T$ and $I_T$ are inversely proportional, when $E_A$ remains constant.

**EXAMPLE:**

$I_T = I_1 + I_2 + I_3$.

If either $I_1$, $I_2$, or $I_3$, is zero amperes, then it is obvious that the parallel resistor that reads zero current is open.