SERIES RESONANT CIRCUITS FILE NO: EP70AS

Resonance is a condition that describes a particular relationship in a series LCR AC circuit. Since resonance is a condition, we will have to describe and analyze that condition

The condition for resonance in a series LCR AC circuit occurs at a frequency which will cause X_L to be equal to X_C.

We can say, then, that in a series circuit containing no resistance, the circuit is at resonance when $X_L - X_C = 0$, thus $X_L = X_C$.

Let's see how this is developed;

X_L

$$X_L = 2\pi FL$$

$$X_C = \frac{1}{2\pi FC}$$

Therefore, $2\ \ FL = \frac{1}{2\pi FC}$

$$\frac{2\pi FL}{1} = \frac{1}{2\pi FC}$$

Cross multiplying

$$4\pi^2 F^2 LC = 1$$

Divide by $4\ {}^2 LC$

$$\frac{4\pi^2 F^2 LC}{4\pi^2 LC} = \frac{1}{4\pi^2 LC}$$

$$\sqrt{F^2} = \sqrt{\frac{1}{4\pi^2 LC}} = \frac{1}{2\pi\sqrt{LC}}$$

$$F = \frac{1}{2\pi\sqrt{LC}}$$

Thus it may be concluded that there is only one frequency at which resonance occurs for a given inductance (L) and capacitance (C).

Next we must look at how this type of circuit may be used and what some of the limits are

In FIGURE 3, we have shown a series LCR circuit with the output taken across the resistor (R).

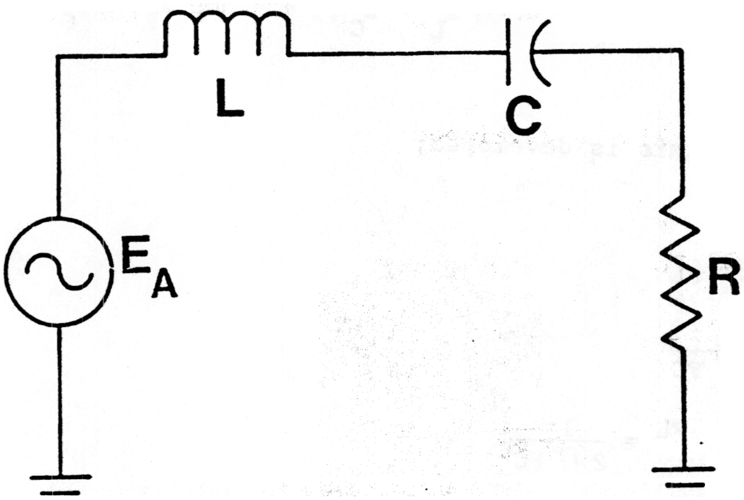

FIGURE 3

The resistor (R) in FIGURE 3 is used to develop the output. We must remember that in the series circuit there is only one path for current flow.

FIGURE 4, shows how X_L and X_C are graphically developed. Remember that the curves on the graph represent the equation for X_L and X_C.

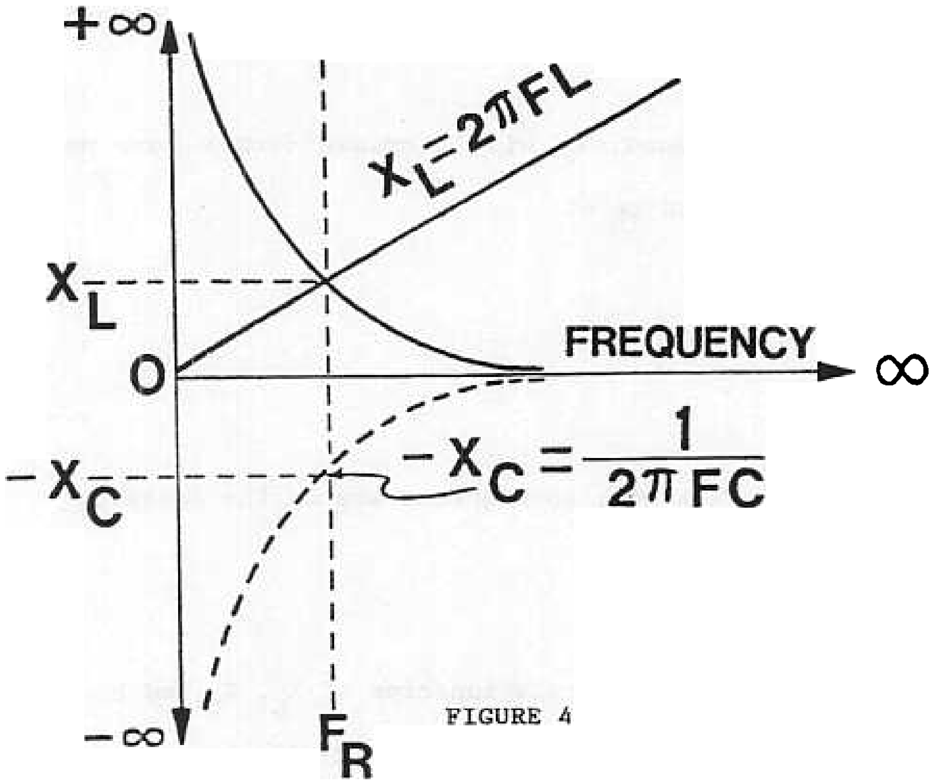

FIGURE 4

You will notice that X_L is a straight line, but X_C is not. X_L is a linear function and X_C is a reciprocal function. You will notice that as frequency increases X_L increases and X_C decreases. There is a point, however, where $X_L = X_C$, and this is at resonance (F_R).

If we examine X_L and X_C in FIGURE 4, it will be noted that as frequency increases X_L will increase above zero

EXAMPLE:

$$X_L = 2 \pi F L$$

This is a direct relationship; both arrows are in the same direction.

If the frequency is increased, X_C will decrease from a more negative value to less negative or toward zero

EXAMPLE:

$$X_C = \frac{1}{2 \pi FC}$$

This is an inverse relationship; both arrows are in the opposite direction.

In FIGURE 5, we can see that the relationships of X_L, X_C and R shown vectorially, X_L is positive and X_C is negative because they are 180° out of phase.

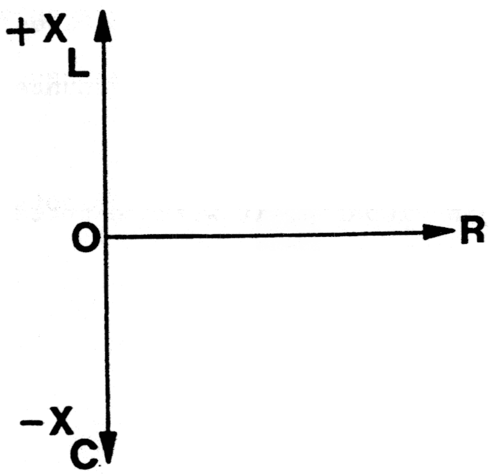

FIGURE 5

It can be seen that $X_L - X_C = 0$, at resonance. $X_L = X_C$ and are at opposite polarities. This can be expressed in a mathematical formula called impedance (Z)

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The term $(X_L - X_C)$ will become zero at resonance, thus $Z = R$ at resonance.

FIGURE 6 shows the impedance curve when Z is plotted graphically against frequency.

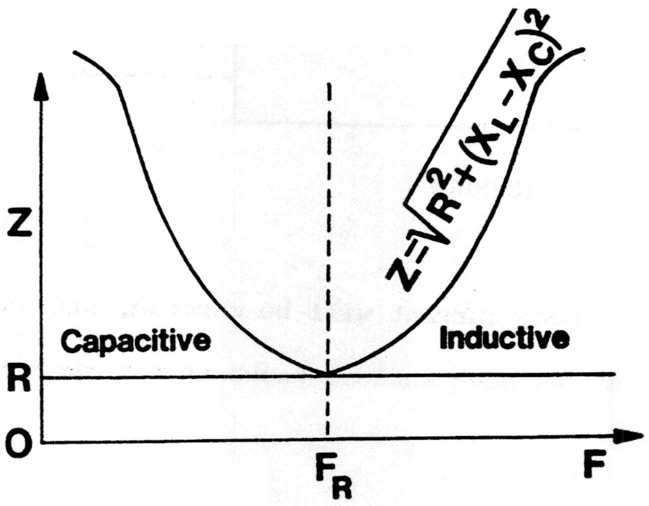

FIGURE 6

FIGURE 6 shows that Z increases as the frequency goes above and below resonance. When the frequency is below resonance, the circuit is basically capacitive; when above resonance, its impedance is basically inductive.

At resonance the impedance is minimum; thus by Ohm's Law $I = \frac{E}{R}$, current will be maximum or limited only by the resistance.

Since this is true and $X_L = X_C$ or $X_L - X_C = 0$, by Ohm's Law, the voltage across the resonant circuit, L and C will be zero (0). We may say the LC circuit is shorted as shown in FIGURE 7.

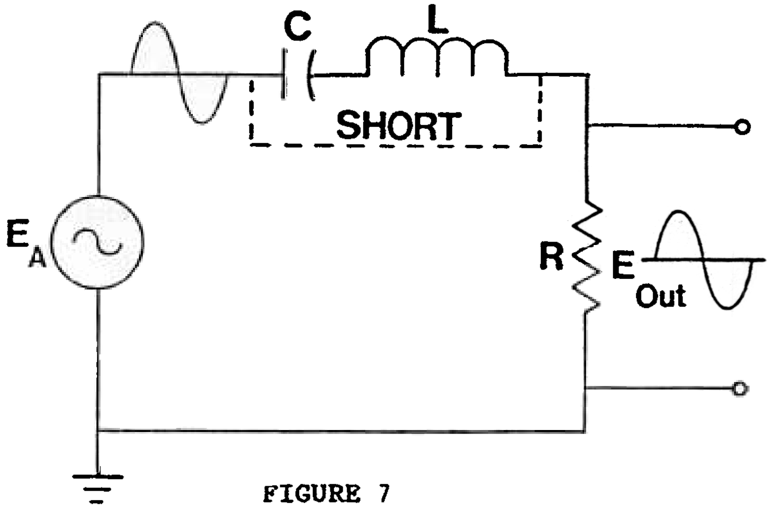

FIGURE 7

If C and L are shorted, current will be maximum, and voltage across R will be maximum; thus we have what is known as a series resonant PASS FILTER.

In FIGURE 8, we can see this voltage , $E_A = \sqrt{E_R^2 + (E_L - E_C)^2}$, plotted graphically against frequency.

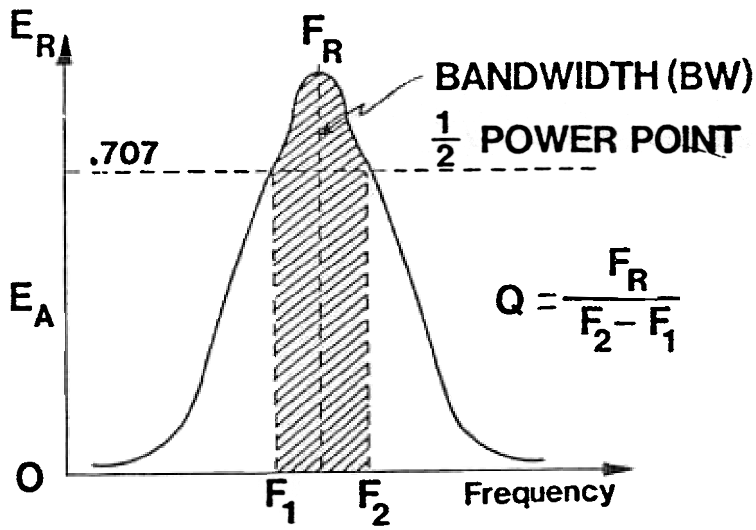

FIGURE 8

FIGURE 8 indicates that at resonance (F_R) the voltage across the resistor in FIGURE 7 is maximum. There are several conditions that can be observed: First, at resonance the voltage across R is maximum.

Second, if the frequency is lowered until the voltage across R is 0.707 x E_{RMAX}, then the half power point is reached at the frequency (F_1).

Third, if the frequency is increased above resonance until the voltage across R is 0.707 x E_{RMAX}, then the half power point is reached at frequency (F_2).

Fourth, the frequency at F_1 describes the lower bandpass frequency and the circuit acts capacitively.

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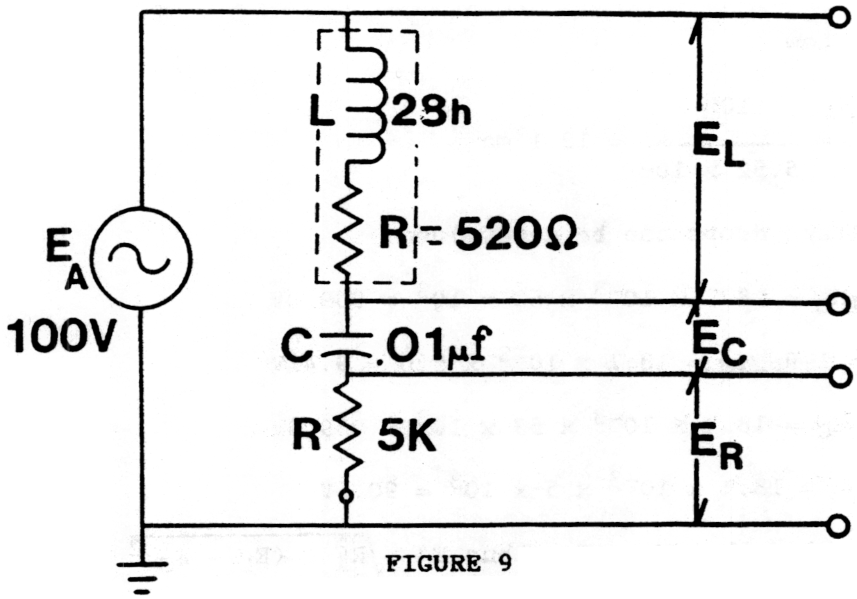

FIGURE 9

FIGURE 9 shows the components that must be considered in calculating the values in the series resonant circuit

From FIGURE 9;

$$F_R = \frac{1}{2\pi LC} = \frac{1}{6.28\ 28 \times .01 \times 10^{-6}} = \frac{1}{3.323 \times 10^{-3}}$$

$F_R = 300\ H_Z$

$X_L = 2\pi FL = 6.28 \times 300 \times 28 = 52.752K$

$$X_C = \frac{1}{2\pi FC} = \frac{1}{6.28 \times 300 \times .01 \times 10^{-3}} = 53{,}079K$$

Basically $X_L = X_C = 53K$

Calculating the impedance:

$Z = \sqrt{(E_R + E_{R-IN})^2 + (X_L - X_C)^2}$ $\quad X_L - X_C = 0$

$Z = \sqrt{(5 \times 10^3 + 520)^2} = \sqrt{(5.52 \times 10^3)^2} = 5.52K$

Using Ohm's Law

$$I_T = \frac{E_A}{Z} = \frac{100}{5.52 \times 10^3} = 18.1 \text{ ma}$$

Now the voltage drops can be determined

$$E_L = I_T X_L = 18.1 \times 10^{-3} \times 53 \times 10^3 = 959.3\text{V}$$

$$E_{R-IN} = I_T R_{R-IN} = 18.1 \times 10^{-3} \times 520 = 9.41\text{V}$$

$$E_C = I_T X_C = 18.1 \times 10^{-3} \times 53 \times 10^3 = 959.3\text{V}$$

$$E_R = I_T R = 18.1 \times 10^{-3} \times 5 \times 10^3 = 90.5\text{V}$$

It must be noted that $E_L - E_C$, thus $EA = \sqrt{E_R^2 + (E_L - E_C)^2} = E_R$ or $E_R + E_{R-IN} = \sqrt{9.41 + 90.5} = 99.91\text{V}$.

The Q of this current must be calculated by using

$$Q = \frac{F_R}{F_2 - F_1} = \frac{X_L}{R} = \frac{X_C}{R}$$, since we do not know F_1 and F_2 , we must use either

$$Q = \frac{X_L}{R} \text{ or } \frac{X_C}{R} = \frac{53 \times 10^3}{R + R_{-IN}} = \frac{53 \times 10^3}{5.52 \times 10^3} = 9.6$$

As shown in FIGURE 10, we can see the relationships between R X_L, X_C, BW, and Q.

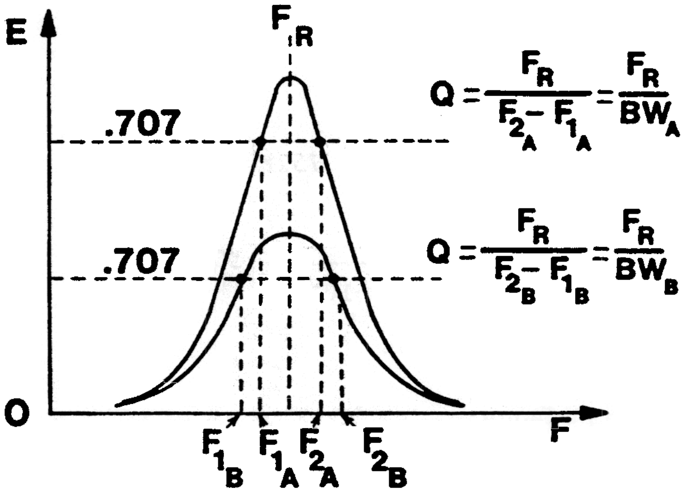

FIGURE 10

Note the bandwidth increases as the Q decreases, even though the resonance and resonant frequency have not changed

It can be seen that the quality (Q) of the circuit can affect the selectivity or the sharpness.

You may have noticed, that when you tune your radio, how rapidly the station volume decreases on either side of the assigned (resonant) frequency

Two things must be noted in this circuit and is typical of series resonant circuits: First, Q or Quality is a unitless number, just 9.0 Second, there are DANGEROUSLY HIGH VOLTAGES developed in some series circuits.

The series resonant circuit is used to pass a band of wanted frequencies. This is basically what is done in tuning a radio or a piece of communication equipment. The output is taken across the resistor, as shown in FIGURE 11.

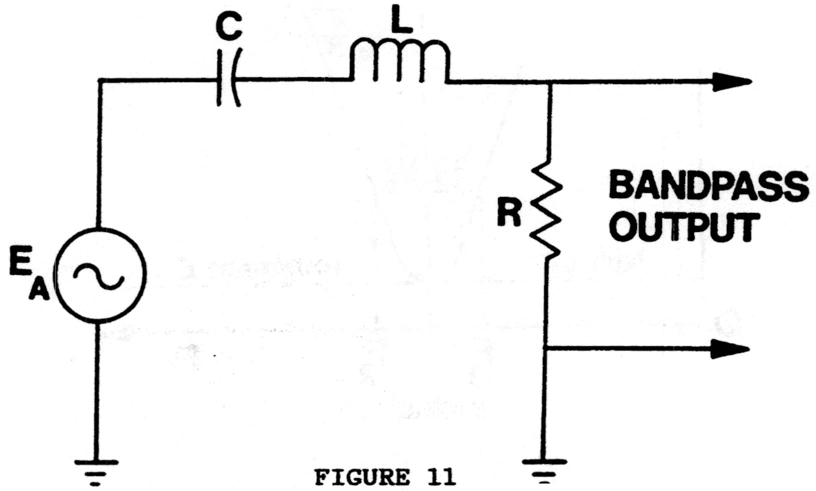

FIGURE 11

The series resonant circuit can be used to reject a band of unwanted frequencies. The type of circuit used to trap unwanted frequencies is shown in FIGURE 12

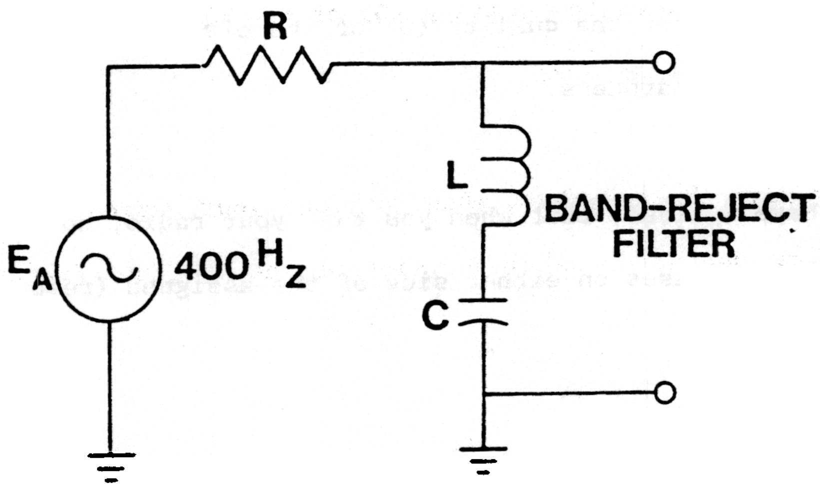

FIGURE 12

From what has been determined, it can be shown that the voltage across L and C of the resonant circuit is zero.

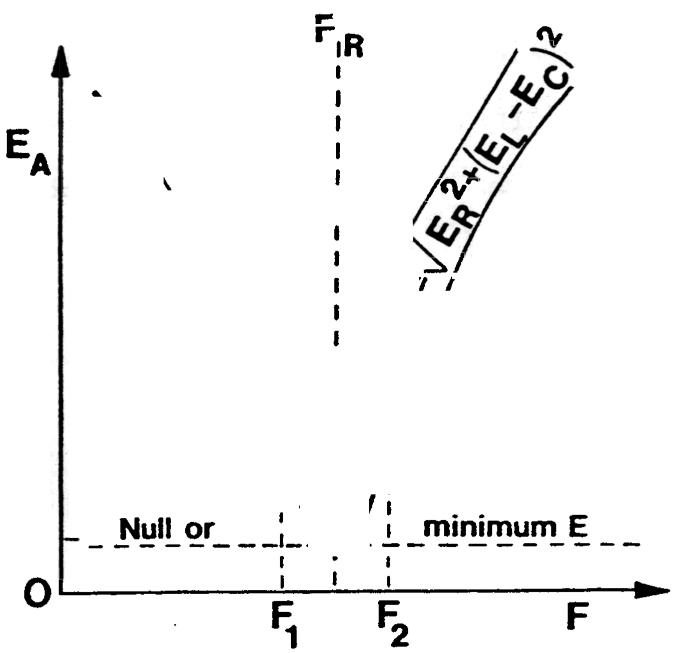

FIGURE 13

band pass and band reject filters are the same circuit. The band pass filter has its output taken across the resistor The band reject filter has its output taken across the series LC Circuit.

calculations for the reject filter will be accomplished the same as in the pass filter.

Impedance (2):

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Q = \frac{X_L}{R} \text{ or } \frac{X_L}{C}; \text{ Or } Q = \frac{F_R}{F_1 - F_2}$$

Bandwidth (BW): $BW = F_2 - F_1$; $BW = \frac{F_R}{Q}$

Lower band pass frequency (F_1):

$$F_1 = F_R - 1/2\ BW$$

Uppwer band pass frequency (F_2):

$$F_2 = F_R + 1/2\ BW$$

To summarize:

A series resonant circuit is basically a pass or a reject filter. For the PASS FILTER, the output is taken across the resistor.

For the REJECT FILTER, the output is taken across the resonant circuit.

The resonant frequency F_R, can be calculated by using

$$F_R = \frac{1}{2\pi \ LC}$$

The reactances can be calculated and must be equal, $X_L = X_C$, at resonance.

The bandwidth is the difference between the frequencies $F_2 - F_1$ or $BW = F_2 - F_1$; $BW = F_R/Q$

The Quality of the circuit (Q) can be determined by two methods;

$$Q = \frac{F_R}{F_2 - F_1}$$

or

$$Q = \frac{X_L}{R} \text{ or } \frac{X_C}{R} \quad \text{since } X_L = X_C$$

d. SUMMARY:

(1 Series resonant circuit analysis:

(a) At resonance

(b) Below resonance

(c) Above resonance

Band Width

Circuit Q.

Pass filters and reject filters